Quincy Course in
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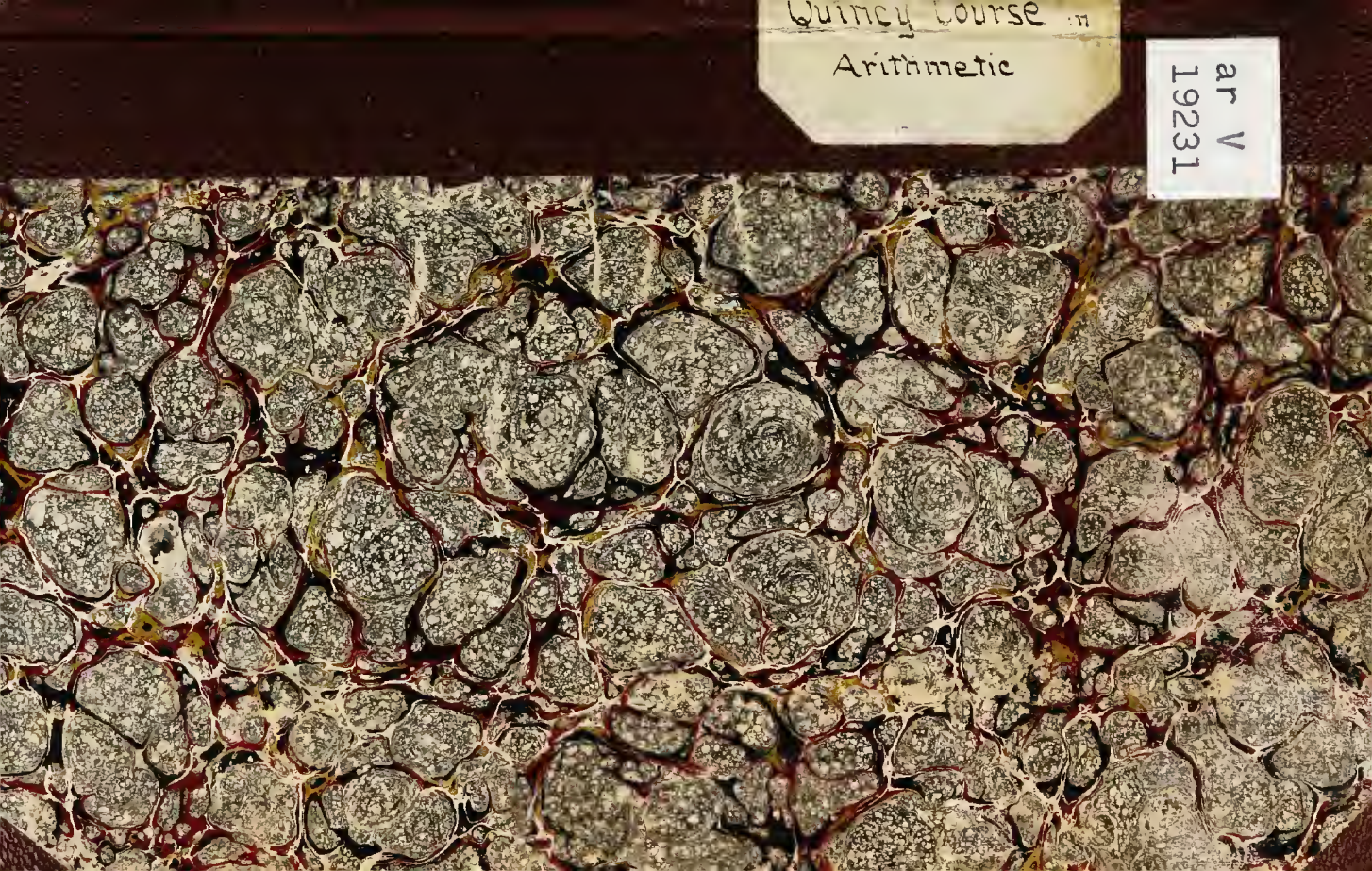

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QUINCY COURSE

—IN—

ARITHMETIC.

BY

COL. FRANCIS W. PARKER,

COOK CO. (ILL.) NORMAL SCHOOL.

CHICAGO:
A. FLANAGAN, PUBLISHER.

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SUGGESTIONS AND DIRECTIONS.

1. A KNOWLEDGE OF NUMBERS and their relations is gained in the same way as a knowledge of color, form, size, or weight. The ideas of which red, square, large, and six are the signs, are taken (abstracted) from objects. What these words (red, etc.) are to the mind depends entirely upon the products of sense-perception, and no amount of study of signs, alone, will ever bring the slightest knowledge of the things they represent. Signs may be learned without a definite association with the ideas that they should recall; they are learned, in fact, and the learning of figures, and not numbers, — signs, not things, — is the fundamental mistake in teaching arithmetic.

2. A NUMBER CAN BE SEPARATED INTO EQUAL OR UNEQUAL PARTS. EQUAL OR UNEQUAL NUMBERS MAY BE COMBINED. NOTHING MORE CAN BE DONE WITH A NUMBER OR NUMBERS.

To illustrate: Present a number of marks, thus, IIII — the sign is 4. Present the same objects thus, — II II The relations *seen* may be expressed: "In four I see two and two," "I see two twos," "Two marks and two marks are four marks," or "Four marks less two marks are two marks." The two relations of separation and combination are *seen together;* they are reciprocal; one suggests the other; one cannot be thought without the other is known, either consciously or unconsciously. Therefore the teaching of one relation is greatly aided by the teaching of the other at the same time.

3. THE COMBINATION OF NUMBERS is ADDITION; of equal numbers is MULTIPLICATION. The separation of a number is SUBTRACTION; the separation of a number into equal parts is DIVISION; the separation of a unit into equal parts gives FRACTIONAL UNITS. *All* that belongs to pure arithmetic has for its foundation, root, and source, the simple putting together and putting apart numbers of things.

4. PLANTS AND NUMBERS SHOULD BE TAUGHT BY PRECISELY THE SAME METHOD. First, the whole plant, or number, is observed; then the parts *on* the plant or number, or severed from the whole; afterward the plant or number is to be compared with all other known plants or numbers.

Grube simply extended the method of object-teaching to the teaching of numbers of objects.

5. THE PROPER WAY OF TEACHING LANGUAGE IN NUMBER PRESENTS A COMPLETE AND LOGICAL ILLUSTRATION OF THE MANNER IN WHICH ALL LANGUAGE SHOULD BE TAUGHT.

Ideas are gained from objects. When an idea becomes by sense-perception a *clear idea*, it *demands* expression. From repeatedly seeing and handling three blocks, three marbles, three sticks, etc., there comes into the mind the idea of which three, 3, or III. is the sign.

The relation of two or more ideas — a thought — is seen again and again. When this thought is clear, it of itself demands expression by a sentence. To illustrate: supposing the ideas and signs, 3, 2, 5, to be already known, then the repeated observation of these objects, III, II, □□□, □□, T T T, T T, * * *, * *, thus arranged, awakens the thought expressed by the sentence, "Three and two are five." The very important law is, teach clear ideas first, then their signs; teach relations of ideas and then the sentences which express them.

We can express that only which is clear in the mind; it is dangerous to *force* expression of that which is dim; lead to clearness before any attempt at expression is demanded. *Ideas grow very slowly, and the most important part of a teacher's duty, after presenting the proper opportunities for the growth of ideas, is to* WAIT *and* WATCH.

Pupils should be allowed to express their thoughts in the idioms which they have been using all their lives. The early introduction of terms, phrases, and sentences entirely foreign to their minds, such as — "divided by," "multiply," "subtract," "taking one number from another," "taking one number so many times," is disastrous; for such forms cannot, for a time, contain a child's thoughts. The child should be slowly led to these expressions by permitting it at first to use its own words and slowly make the new forms known by association and repetition.

The teaching of each number, definition, principle, rule, process, or problem, is a language, as well as an object lesson; in other words, each number, definition, etc., presents an opportunity to teach language. The learning of sentences before the thoughts they express are known robs the child of power to express thought. Great freedom of expression should be allowed, stiff formulas avoided, and the same thought expressed in as many different ways as possible, always, however, leading up to the best possible form of expression.

6. MOST CHILDREN KNOW VERY LITTLE OF NUMBER WHEN THEY ENTER SCHOOL. So far as the writer has investigated this important matter, a large majority of little ones of five years know less than three.

The words they learn in reading are signs of ideas made clear by the incessant sense-activity of five years. They can be readily associated with written words; but the case is entirely different with ideas of number not yet acquired, and that must be learned by the exceedingly slow process of observation. The great evil of learning the signs without the ideas must be avoided, or the result will be ruinous. To learn the ideas and two sets of signs, oral and written, at the same time, seems to be also unscientific, for the multiplicity of dead forms, instead of the ideas of number which make the forms a necessity and give them life, will absorb the child's attention. Following nature's order, oral signs (words) are thoroughly learned first; therefore it is thought best in this course to teach the ideas of number and associate the oral signs alone with them, during the first year. Then, too, when the ideas and their oral signs are firmly fixed in the mind they will give zest and impulse to the teaching of the new (written) forms of expression. Besides, teaching the figures furnishes a capital means of thoroughly reviewing the former work.

7. CULTIVATION OF THE POWER TO REASON, AND THE FORMATION OF THE HABIT OF ACCURATE AND RAPID CALCULATION, ARE THE TWO GREAT MOTIVES IN TEACHING ARITHMETIC.

The first and greater aim is reached by leading to distinct ideas of numbers and their relations; the second by slow, graduated exercises in combination and separation of numbers, adapting each step to the easy grasp of the pupil, and never taking an advance step until all previous ones are mastered; and, above all, never permitting, *if possible, a pupil to make a mistake.*

8. THE FIRST STEPS IN NUMBER SHOULD BE TAKEN WITH GREAT CARE. After the child has been made thoroughly at home in the school-room, the teacher should ascertain by careful and repeated tests just what it knows of numbers. This examination should be made under the most favorable circumstances, and extend over a period of not less than two weeks.

"Bring me so many blocks." The teacher holds up each time the number. "Show me so many." "Touch so many." "Make so many marks upon the blackboard." "Take some blocks in your hand." "How many have you?" This question is the first request for a sign of number. Then may follow the directions, "bring," "show," "touch," "make," three blocks, three marks, etc. "How many hands have you? arms? legs? feet? noses? eyes? ears? mouths? chins?" "How many ——— have I in my hand?" "Now how many?" "Clap your hands three times." "Stamp three times. "Open your mouth three times." "Shut your eyes three times."

These questions indicate something of the way a child's knowledge of number should be tested. The exercises for a time should not be continued more than three minutes.

9. WHEN A CHILD'S KNOWLEDGE OF NUMBER IS ASCERTAINED, BEGIN TO BUILD THERE. Lose no time in trying to teach children what they already know, and never take an advance step until the preceding ones are thoroughly mastered.

10. A GREAT VARIETY OF OBJECTS SHOULD BE USED IN TEACHING NUMBER. For the use of one kind of objects alone leads children to associate their ideas of number with them, while clear ideas are more easily taken (abstracted) from a great many different kinds of objects. Besides, changing from one class of objects to another sustains the interest. It is plainly evident, too, that seeing and handling many classes of objects trains the observing powers to make distinctions and to classify things. See S. 1, Course of Study.

11. UNTIL TEN IS TAUGHT, PUPILS SHOULD HANDLE THE OBJECTS THEMSELVES. Two senses are thus trained, — sight and touch. The plan of causing children to *do everything for themselves* is thus put in practice. "Take two blocks in one hand and three in the other." "Put them together." "How many have you." "Two blocks and three blocks are ——?" "Make three marks, a dot, now make two more marks." "How many marks have you made?" "Five less two are how many?" Let pupils in answering this question take five objects and separate them into two parts, one of which is two.

For tests and reviews the teacher should handle the blocks,— alone, suggesting the questions by different movements with the blocks, thus, — IIIII. "I see five marks" (or five marks simply), III, II. "In five marks I see three marks and two marks." "Five marks less two marks are three marks." Erase the dot, IIIII. "Three marks and two marks are five marks." Show II, II, thus, then put them together, IIII. "Two marks and two marks are four marks." "Say it another way." Ans. — Two twos are four.

Marks on the blackboard should be made large and distinct, generally with colored crayon.

12. THE TEACHER SHOULD THOROUGHLY LEARN TO MAKE ALL THE COMBINATIONS indicated by table (see page 11), and in the order there laid down. Three ways of giving a lesson are suggested. 1st. Pupils standing around a long table plentifully supplied with objects. Let each pupil have a definite number of objects, — selected by the pupils themselves. (It may be well at times for one or two pupils to arrange the blocks for a lesson.) The teacher at the head

of the table rapidly and skilfully directs the movements of the class. 2nd. Class at blackboard, — directed by the teacher, — at blackboard on the opposite side of the room. The ways of using the blackboard in number are innumerable. 3rd. Class in their seats with objects on the desks before them. *The lessons must be short, never exceeding ten minutes during the first year.*

13. THE WRITTEN SIGNS OF NUMBER ARE TO BE LEARNED THE SECOND YEAR. The process of teaching figures is precisely the same as in teaching written words. Show a number of objects, and then write (on blackboard) the sign. Write the sign and ask pupils to show that number of objects. Show a number of objects,—and have pupils write the sign. Class at the board. Show numbers of objects one after the other, and have pupils write the signs. Show III, II, thus, then change, IIIII. "Write that." "Three and two are five." Teacher erases *and* and writes +, *are* and writes =. "Now read it the same way as before." Show objects as in oral teaching and have pupils write the answers.

$7 - 4 = 3.$	"Erase answers."	$7 - 4 =$
$6 + 2 = 8,$		$6 + 2 =$
$9 \div 3 = 3.$		$9 \div 3 =$
3 3's $= 9.$		3 3's $=$
$3 \times 3 = 9.$		$3 \times 3 =$

"Write answers rapidly." "Erase answers again." "Read columns." "Erase second line."

$7 - \quad = 3.$	"Fill up the columns."
$6 + \quad = 8.$	"Erase again."
$9 \div \quad = 3.$	"Read."
3 's $= 9.$	"Erase first line.
$3 \times \quad = 9.$	

Use in these exercises all the forms of stating processes to be found in arithmetical calculation, the pupils learning them by seeing the relations which they express. In division, for example, $6 \div 2 = 3$, $2)6$, 3, $\frac{2)6}{3}$; in multiplication 2 4's $= 8$, $2 \times \quad = 8$, $\frac{4}{2}$. $\frac{}{8}$ When these forms are firmly fixed in the mind, give the same exercises without using objects. From 10 proceed, number by number, to the development of 20, using both oral and written work. For reviews give an exercise like this orally. Have pupils write out answers upon slates or board, in columns, *without hesitation*. $9 + 3$; $6 + 4$; 3's in 12; $12 - 6$; $\frac{1}{2}$ of 8; 7×2. Let pupils change slates and correct,

the teacher reading the answers. All means of training children to read numbers at sight — i.e., add, subtract — should be used. *Avoid forming the habit of hesitating.*

14. REASONING IS SEEING THE RELATION OF THINGS. A problem represents things as being in certain relations to each other; the solution of the problem depends primarily upon the mind's comprehension of the things represented and their relations. In other words, the mind must grasp the exact conditions stated in the problem, as the first step to its solution.

15. THE HABIT OF TRYING TO PERFORM THE PROBLEMS WITH THE FIGURES ALONE, WITHOUT ATTEMPTING TO UNDERSTAND THE CONDITIONS, IS FATAL TO ALL PROGRESS AND COMPLETELY BARREN OF GOOD RESULTS.

The proper use of objects is the first step in forming this all-important habit of *thinking* (seeing relations of things), but at every step during the whole course, when pupils cannot mentally picture out the things represented, recourse should be had to objects. See S. 14.

16. PUPILS SHOULD BE TRAINED TO ILLUSTRATE EVERY PROBLEM WHEN POSSIBLE BY DRAWING AND OBJECTS. It is better to perform one problem understandingly than a hundred that are only partially understood. *Quality, not quantity.* See S. 17. Explanations by the teacher are entirely unnecessary, if the proper occasions are presented for the mind to act.

It is a good plan to write a problem upon the blackboard, have a pupil read it, then, with the close attention of the entire class, ascertain, by sharp questioning, if they understand it. Follow this by asking a pupil (generally the dullest) to take the first step in performing the problem; ask another, and another, to take the first step; when satisfactory, have the work written upon the board, and then take the next step in the same way, and so on to the end.

It is also a good plan to cause the work of a problem to be written out upon the board before any calculation is made, thus: —

"How many apples at 2 cts. apiece can you buy for 8 oranges at 4 cts. apiece?"

Work written: $\frac{8 \times 4 \text{ cts.}}{2 \text{ cts.}} =$

17. See 12, *a*. EACH DEFINITION, RULE, PROCESS AND PRINCIPLE, AS IT OCCURS IN TEACHING, PRESENTS AN EXCELLENT OPPORTUNITY FOR AN OBJECT AND LANGUAGE LESSON. A definition is a description of a thing, or things in their relations, that can be seen. A rule is a description of a process that can be placed before the pupil's

eyes; a principle can be discovered in a process. When the proper time comes (beginning of the fifth year in the course) this interesting series of object and language lessons should begin and be kept up through the entire course.

SUBJECTS FOR OBJECT AND LANGUAGE LESSONS.

DEFINITIONS.	RULES FOR
A Number.	Addition.
Addition.	Multiplication.
Multiplication.	Subtraction.
Subtraction.	Division.
Divison.	
Notation.	Principles of Notation.
Numeration.	
Fractions.	

These are a few of the subjects that should be used to test a pupil's power of seeing and describing things and processes.

LESSON UPON THE DEFINITION OF A NUMBER.

Hold up a number of objects, another and another of different objects. "What is this?" "What are these?" "It is a lot of blocks." "Five shells." "A number of things." "Is this a number?" "Yes." If this is a number, what is a number?" (showing a number of objects.) "It is a lot of things together." "It is several things." "It is many things." "Do not say 'lot' or 'several.'" "I am putting down a number of things upon the desk, now I take them up one by one. What did I do?" "You took them up one by one." "Do not say, 'you *took them up*.' Say you ——— 'collected' them." "What are they together after I have collected them." "A collection." "A number." "Take your slates and write what a number is." Each pupil should read his definition; the best (if correct) should be taken as a model and written by all.

LESSON UPON THE RULE FOR ADDITION.

"We will try to find the best way to add numbers upon the slate or blackboard." Teacher writes numbers on different parts of the board. "Can we add these figures as they are?" "No." "Yes." "No, we cannot." "Try it." "Yes, we can add them, but it is not the best way." "What is the best way?" "Write them together." Teacher writes the numbers together, but does not put units under units, etc. "That is not the best way." "Please take your slates and write the best way of writing numbers to add." Thus each step in the process can be found out and written by the pupils.

In "carrying," "borrowing," the processes of long multiplication and division, "the best way" should be discovered by the pupils. See Fifth Year, T. R. *a*.

18. AFTER THE FUNDAMENTAL WORK HAS BEEN MASTERED, NOTHING NEW CAN BE INTRODUCED; EVERYTHING CAN AND SHOULD BE REFERRED DIRECTLY BACK TO THE SIMPLE ACTS OF COMBINATION AND SEPARATION.

Terms "factor," "principal," "interest," "base," etc., are simply old, well-known friends under new names.

FACTS TO BE TAUGHT WITH TEN OBJECTS.

ARRANGED IN ORDER OF TEACHING.*

(1) $1-1$. (2) 2×1, 1×2, $2\div1$, $2\div2$, $1+1$, $2-1$, $2-2$, $\frac{1}{2}$ of 2, $\frac{2}{2}$ of 2. (3) 1×3, 3×1, $3\div3$, $3\div1$, $2+1$, $1+2$, $3-1$, $3-2$, $3-3$, $\frac{1}{3}$ of 3, $\frac{2}{3}$ of 3, $\frac{3}{3}$ of 3. (4) 4×1, 1×4, $4\div1$, $4\div4$, $1+3$, $3+1$, $4-3$, $4-1$, $2+2$, 2×2, $4-2$, $4\div2$, $4-4$, $\frac{1}{4}$ of 4, $\frac{2}{4}$ of 4, $\frac{3}{4}$ of 4, $\frac{4}{4}$ of 4, $\frac{1}{2}$ of 4, $\frac{2}{2}$ of 4. (5) 1×5, 5×1, $5\div5$, $5\div1$, $4+1$, $1+4$, $5-1$, $5-4$, $2+3$, $3+2$, $5-3$, $5-2$, $5-5$, $\frac{1}{5}$ of 5, $\frac{2}{5}$ of 5, $\frac{3}{5}$ of 5, $\frac{4}{5}$ of 5, $\frac{5}{5}$ of 5. (6) 6×1, 1×6, $6\div1$, $6\div6$, $1+5$, $5+1$, $6-5$, $6-1$, $4+2$, $2+4$, $6-2$, $6-4$, 2×3, 3×2, $6\div2$, $6\div3$, $3+3$, $6-3$, $6-6$, $\frac{1}{6}$ of 6, $\frac{2}{6}$ of 6, $\frac{3}{6}$ of 6, $\frac{4}{6}$ of 6, $\frac{5}{6}$ of 6, $\frac{6}{6}$ of 6, $\frac{1}{2}$ of 6, $\frac{2}{2}$ of 6, $\frac{3}{3}$ of 6, $\frac{2}{3}$ of 6, $\frac{3}{3}$ of 6. (7) 1×7, 7×1, $7\div7$, $7\div1$, $6+1$, $1+6$, $7-1$, $7-6$, $2+5$, $5+2$, $7-5$, $7-2$, $4+3$, $3+4$, $7-3$, $7-4$, $7-7$, $\frac{1}{7}$ of 7, $\frac{2}{7}$ of 7, $\frac{3}{7}$ of 7, $\frac{4}{7}$ of 7, $\frac{5}{7}$ of 7, $\frac{6}{7}$ of 7, $\frac{7}{7}$ of 7. (8) 8×1, 1×8, $8\div1$, $8\div8$, $1+7$, $7+1$, $8-7$, $8-1$, $6+2$, $2+6$, $8-2$, $8-6$, $3+5$, $5+3$, $8-5$, $8-3$, 2×4, 4×2, $8\div2$, $8\div4$, $4+4$, $8-4$, $8-8$, $\frac{1}{8}$ of 8, $\frac{2}{8}$ of 8, $\frac{3}{8}$ of 8, $\frac{4}{8}$ of 8, $\frac{5}{8}$ of 8, $\frac{6}{8}$ of 8. $\frac{7}{8}$ of 8, $\frac{8}{8}$ of 8, $\frac{1}{2}$ of 8, $\frac{2}{2}$ of 8, $\frac{1}{4}$ of 8, $\frac{2}{4}$ of 8, $\frac{3}{4}$ of 8, $\frac{4}{4}$ of 8. (9) 1×9, 9×1, $9\div9$, $9\div1$, $8+1$, $1+8$, $9-1$, $9-8$, $2+7$, $7+2$, $9-7$, $9-2$, $6+3$, $3+6$, $9-3$, $9-6$, $4+5$, $5+4$, $9-5$, $9-4$, 3×3, $9\div3$, $9-9$, $\frac{1}{9}$ of 9, $\frac{2}{9}$ of 9, $\frac{3}{9}$ of 9, $\frac{4}{9}$ of 9, $\frac{5}{9}$ of 9, $\frac{6}{9}$ of 9, $\frac{7}{9}$ of 9, $\frac{8}{9}$ of 9, $\frac{9}{9}$ of 9, $\frac{1}{3}$ of 9, $\frac{2}{3}$ of 9, $\frac{3}{3}$ of 9. (10) 10×1, 1×10, $10\div1$, $10\div10$, $9+1$, $1+9$, $10-1$, $10-9$, $3+7$, $7+3$, $10-8$, $10-3$, $7+3$, $3+7$, $10-4$, $10-6$, $4+6$, $6+4$, $10-3$, $10-7$, $5+5$, $10-5$, $10\div2$, $10\div5$, 2×5, $5+2$, $10-10$, $\frac{1}{10}$ of 10, etc., $\frac{1}{2}$ of 10, $\frac{2}{2}$ of 10, $\frac{1}{5}$ of 10, $\frac{2}{5}$ of 10, $\frac{3}{5}$ of 10, $\frac{4}{5}$ of 10, $\frac{5}{5}$ of 10, $3=$ one two and 1, $5=$ two twos and one, $7=$ two threes and 1, $9=$ two fours and 1, etc.

* This arrangement is nearly the same as the First Number Card published by Augustus D. Small, Supt. Schools, Salem, Mass., who has kindly given his consent to its partial reproduction here.

QUESTIONS SUGGESTING THE METHOD OF DEVELOPING NUMBER.

Taken largely from Horace Grant's Arithmetic for Young Children.

Operations with Objects to 4.

Let the teacher show three blocks, saying to the child, "Bring me as many blocks from the table; as many pebbles; make as many marks upon the blackboard," etc. Let the teacher take up a block, saying, "This is one block." Let the child be told to put one block on the table; one pebble, one bean, etc. The teacher should then speak to the pupil as follows:—

"Show me one finger. Show me one chair. How many heads have you? How many noses have you? I have placed two blocks on the table; now I have taken them off. Try if you can put two blocks on the table, two beans; etc. Put away the two blocks, and put out two shells. Take away one shell, and how many are left? Take away the other shell; how many are left? Put out two horse-chestnuts; say 'one' for every horse-chestnut that you have put out. Clap your hands once. Show me two fingers; show me two thumbs. Take two steps on the floor; go backwards one step. How many mouths have you? You have one head what else have you one of? Tell me all the things you have two of. Here is one block and one block; what are one block and one block called? Hold up one hand, and also the other hand; how many have you? Put down one; how many are up now? Clap your hands twice or two times." ('Twice' and 'two times' should be used alternately, and in such a manner that the child shall perceive that they mean the same things.)

"Look at what I am doing: I have put out three pebbles, and now I have put three back. Put out two sticks, now put out another; how many have you put out? Say 'one' for each stick you have put out; how many times have you said 'one'? Hold out three fingers. Point to three children; to three boys; to three girls; to three teachers. Show me three legs of a chair. Strike the table once and once; how many times have you struck it? Shut your hand; open one finger; open another finger; how many are open? Open another finger; how many are open now? Lay down one splint and one bead; how many things have you laid down? Lift up your foot three times. How many plates at dinner must be put out for you and me? Take three steps forward and two steps backward. How many joints has your thumb? How many joints has your forefinger? How many joints has your forefinger more than your thumb? I have put one cent upon the table; put as many beside as shall make three altogether? Nod your head twice and once; how many times have you nodded? Stamp your foot twice and once; jump twice and once; take two and one steps. Make a mark on the blackboard for every window in this room." (Pupils may be allowed to draw all the windows.)

"Draw a square and put three birds in it. Draw an apple and put three seeds in it. Make a mark for each door in the room. Draw

each door in the room. Draw a boy going out of each door. Make three squares on the blackboard; three boxes; three lines, etc. Clap your hands once and once and once; how many times have you clapped them. Put three slates on the table; now put another; three slates and one slate are four slates. Put three nuts and one nut on the table; how many nuts have you placed there? Put out two pebbles; now two more; how many have you put out altogether? Say 'one' for every pebble you have put out. Make four marks on the blackboard. Bring me four blocks. How many blocks have you brought me? How many ones? how many twos? how many fours? Carry one back; how many have you left. Shut your hand; open one finger; open two other fingers; how many are open? Put down as many splints as you have hands and mouths. Show me one chair and one table? How many things have you shown me? how many tables? how many chairs? Show four fingers; touch four of my fingers. How many are you and I? How many are we if we count John and Mary? Put on the table two blocks; make a mark for this one, now make a mark for the other; How many marks have you made? Rub out one of the marks; how many marks are there now? Rub out another; how many have you now? How many have you rubbed out altogether? Here are four pictures; how many ones do you see?

"Walk four steps; jump four times; hop on one foot four times; tap the table four times. Tell some little stories about four things. Who can see four things in this room? Now, John, tell me the four things you see. James; Sarah. How many things have I altogether in my pocket, if I have a pencil, a knife, and a key? Put out two blocks; now put out two more; how many twos have you put out? Five twos are called what? Make a mark for each leg of this chair; how many marks have you made? Put four pebbles in a row. Put four pebbles in two rows; how many are in each of these rows?

"How many fingers have you on your right hand without counting your thumb and little finger. How many have you if you count your thumb? Say your own name three times. Put down two blocks; put down two more; now take away one block; how many are left? If John and you and I had one cent each, how many cents should we all have? How many legs has this cat? (Showing picture.) How many legs has the cat more than you have? If you had one leg more should you have as many legs as the cat? How many legs have you less than the cat? Take out three beans. Take one of three beans, how many are left? Put out one block; put two in a row below that; put three in a row below that; put four in a row below that; make as many marks, as three ones in three; rub out one mark; how many marks are left? Rub out another mark; how many are now left? How many handles have three knives? Take two blocks yourself, and give as many to me as will make four altogether. Hand me one block with your eyes shut; three blocks; four blocks."

Operations with Objects to 6.

"Bring me four pebbles; now bring me one more; four pebbles and one pebble are called five pebbles. Put out five blocks; five shells; five beans; make five marks; stamp five times; show me five boys; five girls; five children. Open your hand wide; how many fingers

are open (with the thumb)? Show your hand; how many fingers are open? how many are shut? Put out five blocks; put them in two rows; how many are in each row. Put them in three rows; how many are in each row. Open your hand; then shut one finger; how many fingers are opened? shut three fingers; how many are left open? How many are left open when you shut two fingers? when you shut four fingers.

"Try how many ways you can arrange three blocks. ∴ ⁝ ∴ ∴ etc.

"If I buy two cakes at one cent each, how much shall I pay the baker? Here is a five cent piece, and there are some cents; put down as many cents as would buy as much as this five cent piece. Show me a couple of fingers; show me two couple of fingers. ('Couple' should be explained if not understood by the child.) How many fingers have you altogether on your right hand? on your left hand? W^at else have you five of? How many panes of glass are there in the lowest row of panes in the window? How many more do I need to make up five panes? How many ducks are a couple of ducks? Has this chair more legs or fewer legs than five? How many fewer? Are your arms and mine five? How many arms have you less than four? How many less than five? Take two blocks for yourself, take one for John, and give me as many as will make us have five among us. If I were to give two apples for you and John, how many should you have and how many should he have?

"If John had two cents and you and I had a cent each, how much would all of us have. If you have three cakes to divide between yourself and your sister and me, how many should each of us have? Shut your eyes and take up three from these five counters. If you had one and two nuts in your right hand, and two and one nuts in your left hand, which hand would hold most nuts?

"This stick, or measure, is one foot long; this other stick, or measure, is one yard long; show me how far a foot goes on the yard measure. Try if you can find how many feet are as long as a yard. Measure this chair. Is it a foot broad? How many feet broad is it? Measure this table. Is it a yard high? Is it yard across? Measure the door with the yard measure. Is it a yard? Measure it with a foot measure. How many feet wide is it? Measure two yards along the floor beginning at the door. Measure three feet in the same manner. Try if you can measure the length of a yard on the floor with a foot measure. Draw a line a foot long on the blackboard. Now take your measure and see if you are right. John, James and Abby may draw a line one yard long on the blackboard. Frank Gilbert may take the measure and see who has drawn the best. Guess how long your desk is; take the measure and see if you guessed right. Draw a triangle; how many lines or sides are there? If this side were a foot long, and the other sides were each of the same length, how many feet would all the sides together measure? When I say 'one, one,' how many words do I say? when I say 'two,' how many words do I say? 'Two' is a short way of saying what? What would two things be called if one were taken away. What would 'one' be called if another were added or put to it. Take up five blocks; now take up another block; five blocks and one block are called six blocks. Show me six shells; six beans; six slates; make six marks,—say six words. Take six beans and put them in twos; how many twos are there? Put them in threes; how many threes are

there? Put them into ones; how many ones are there? How many fours can you find in six shoe pegs? How many fives? In six shoe pegs how many threes do we find? How many twos?

"Write six words on your slates. Write six rows of i's — each row containing six i's.

"If I cut this square piece of paper into two parts of the same size, what is each part called? How many halves is it cut into? Here is a splint which is cut into two parts of the same size; what is each one of these parts called? How many halves are there in the whole splint? Here is an apple that is not cut; how many halves could it be cut into. Draw a line; divide it into two parts of the same length; what is each part called? Half of what? The whole line is made up of how many halves? If I gave you one apple between yourself and Philip, how much of it ought you to keep, and how much of it should you give to him.

"This weight is called a pound weight. Take it in your hand. A piece of bread or a stone, or anything just as heavy as this, would weigh a pound, or be a pound weight. This weight is half a pound. Take it in your hand. How many of those weights do you think would weigh as much as a pound weight. (A number of very entertaining exercises, similar to those with the yard measure, may be performed by the child with a small pair of scales and a few weights.) A girl carried a pound of sugar in one hand, and two pounds of sugar in the other; how many pounds of sugar did she carry. If a package weighs twice as much as the pound weight, how much would it weigh? Suppose that you picked up a stone that is half as heavy as this pound weight, how much would it weigh. How many such stones would weigh a pound? A man went to market and bought a pound of meat, two pounds of bread, and a pound of butter; how many pounds had he to carry home in his basket?

"A farmer had two sheep, each of which had two little lambs; how many lambs were there? Another farmer had two sheep and three lambs; one of the sheep had one lamb only; how many lambs must the other sheep have had?

"Try in how many ways you can arrange four blocks. Put out two shoe pegs; take away half. Put out twice as many pebbles as one. Put down four blocks; take take away half. Put out half as many pebbles as four. Arrange six blocks in pairs; how many pairs do you find. Put down six shells. If one shell be taken out of six, how many remain? if two be taken? how many are left if three are taken? If once two be taken from six shells, how many are left? if two twos or twice two? if three twos or three times two? How many things are three chairs, two lamps and a fiddle? Try if you can find out without looking, how many horse-chestnuts there are in each of my hands. A hen had six chickens, but some rats killed two of them; how many chickens had she then left of six. Another hen had also six chickens, and some rats ate one, and two fell into a ditch and were drowned; how many chickens had this hen left."

Operations with Objects to 7.

"Tell me another name for one and one. Tell me another name for one, one and one. Twice one, or two times one, is called what?

"Make a square; how many sides has this square? how many

lines? how many corners? If the top line is a yard long, how many yards long would all the sides measure? Divide the square into two equal parts; how many parts did one-half make? two halves? Some men put a post into the water to tie boats to; the post was three feet under water, and two feet above water; how long was the post? If two feet were under water and two feet above water, how long would it be?

"How many legs must be put on a dog for him to have five legs?

"A little insect, that had six legs, met with an accident and had two broken off; how many legs was it obliged to walk with afterwards? How many feet have you and Willie Worster and I? How many hands? how many noses? Three old soldiers were walking along the road; William had but one leg, Charles had two legs, and John only one; How many legs had these three soldiers to walk with? How many wooden legs would they need, so that each soldier should have two legs? Make six marks of any kind that you like. How many are left if you rub out two and one? Are any left if you rub out three and two? What number is the half of two? What number is twice two or two twos? Cut this bit of paper into two parts of the same size; what is each of these parts called? If an orange were divided equally between you and George, what part would each of you have? If a sailor had half as many hands as you, how many would he have?

"This can holds a quart and this dish holds a pint; now if I fill the dish with water and pour it into the can, the can would be half full; what must I do to fill the can. How many pints in a quart? In a quart how many pints? What part of a quart is a pint? One is what part of two? A spider caught in his web, one fine day, two flies and a bug and a bee; how many insects did he catch that day? how many animals? How many did he eat for dinner if he ate all but the bee? John took two buttons off his jacket, and Harry lost twice as many; how many did John and Harry lose together? If your cat had half as many legs as she has now, how many would she have?

"Make six marks; make five under them; four under them; three under these; two under these; one under these. How long must I keep a puppy that is a month old now before he is six months old? If I were to place six nuts on the table and tell you to take one-third of them, how many would you take?

"I have five hens and a duck; how many hens have I? how many ducks? how many fowls? If I were to divide a quart of cherries into two equal parts, what would each of those parts be called? What part of a quart is a pint? Two pints are how many halves of a quart? How much is two added to one more than one added to two? Here are five pebbles and there are six pebbles; which is the larger number. How much larger?

"Put out six splints; now add another to them; six splints and one make seven splints. Show me seven shells; seven shoe pegs; seven boys; seven girls; seven fingers; make seven marks.

"Write seven words; write i seven times; take seven steps; clap your hand seven times; go into the entry and rap on the door seven times; put blocks on seven desks; shake hands with seven children. How many days are there in a week? Put out seven blocks; how many twos can you find in them? how many threes; how many fours? how many fives? how many sixes? how many sevens? Show me

six pebbles; how many shall you add to them to make seven? Show me four blocks; how many more will make seven. Five splints and how many are seven splints? Three boys and how many are seven boys? Have two chairs seven legs altogether or more than seven. Here are a five cent piece and a two cent piece; how many cents in all? How many single stockings are there in a pair? in two pairs? A man had six chickens, sold two, lost one and ate two; had he any left? A house had three windows in front, and three behind; had it more or less than seven windows altogether?

"A man had four books, and afterwards bought as many as with the four made seven; how many did he buy? How many times must I empty a quart jug into a tub to have seven quarts of water? A gallon is four quarts; how many quarts is half a gallon? Draw a triangle; how many sides has one triangle? how many corners? Ask the same of two triangles. Make the number of marks which are one less than seven; two less than seven; four less than seven. How many eyes must you have to have seven. A hen laid seven eggs in two months; the first month she laid three eggs; how many did she lay the second month? Another hen laid seven eggs in three weeks; one week she laid three, the next she laid two; how many did she lay the third week? There are three windows in a room, and each required two yards of holland for a curtain; how much holland must be bought for all the curtains? Put all the numbers you know upon the table.

"Name the days of the week. Write their names. How many are there? Four weeks are as much time as one month; how many weeks are there in half a month? How many weeks are there in a month and a half?

"I have cut this square piece of paper into four equal parts; what is one of the parts called? Look at this splint; it has been cut into four; what is each part called? how many fourths or quarters are there in the whole splint? Here is a foot measure; show me half a foot; show me a quarter of a foot. Draw a square, each side of which is a foot long; now divide the square into four equal parts; how many squares can you see? how long are the sides of the little squares? Draw a line; divide it into halves; divide it now into quarters; how many quarters did you divide each half into?

"Draw four horizontal lines, and three vertical lines; how many lines have you drawn? Draw three slanting lines, two vertical lines, and two horizontal lines; how many lines have you drawn? Draw a tree; put five birds on one limb, and two birds on the ground close to the tree; how many birds on the tree? Each day I read a page in my new book; how many pages do I read in a week?

"Here are five blocks and seven blocks; which is the larger number? Write seven rows of i's on your slate, with seven i's in each row."

Operations with Objects to 8.

"A quart is two pints; how many quarts are there in four pints? in three pints? in seven pints? Do you recollect how many feet there are in a yard? how many feet are there in two yards? in a yard and one-third? How many yards are there in six feet? in seven feet? in four feet? Draw a square one-half yard long on each side; how long are the four lines? Put down four blocks; take away one-quar-

ter of them; how many are left? Four times what are equal to four? What is the half of four? what is the quarter of four? Mr. Spear walked two miles in an hour, and his son walked four miles in an hour; how much faster did the son walk than his father? It would take Mr. Spear three hours to walk from here to Boston; how many miles is it to Boston? How long would it take Mr. Spear's son to walk to Boston? Seven cubes and one cube are called eight cubes. Show me eight shells; how many fours can you find in them? how many twos? how many eights?

"Take eight steps; show me eight fingers, eight boys, eight girls; make eight squares, eight triangles; write eight words; write the letter i eight times; draw a line, and divide it into eight parts. Give each one of these children eight beans; now take away half of eight beans from each one; how many has each left? Eight pebbles mean the same thing as twice how many?

"How many legs have a man and a horse? how many legs have two horses? A careless little girl lost a needle every day for a week, but found two of the lost needles; how many were not found? Four women bought a pound of tea; how much must each have for her share when the tea is divided? Take eight marbles; can you separate them into two equal parts? into three equal parts? into four equal parts? into five? into six? into seven? into eight? Eight is another way of saying four times how many? I will cut this square of paper into three equal parts; what is each part called? How many thirds are there in the whole square? What part of a yard is a foot? What part of a yard is two feet? How many lame legs must two dogs have between them if they have seven sound legs? A dog was run over by a wagon, and one leg was hurt; how many legs had the dog then? Can you tell me two numbers that are equal to eight?

"A young apple-tree had upon it the first year one apple; next year twice as many; in the third year twice as many as on the second; how many apples did the tree produce in three years? How many thirds are there in one apple? in two apples? in one apple and a third? How many sides (faces) has this cube? Take eight shells; separate them into four equal parts; what number will each of these parts be? what part of eight will it be?

"If three boys had seven apples to divide among them, and one boy took three, how many apples would be left for each of the two other boys? One hen had eight chickens; another hen had half that number; how many had she?

"A snail climbed up two feet of a wall every day, but slipped back one foot every night; how many feet did he get up in three days? Take these blocks, and show me all the numbers that you know. Two and what are equal to five? Can you find two equal numbers that are equal to five?"

Operations with Objects to 9.

"Put down eight blocks and one block; eight blocks and one are called nine. Put these nine blocks back, and put down nine pebbles. Seven pebbles and how many are nine pebbles? Into how many threes can you separate these nine pebbles? Try how many lots of four cubes there are in nine cubes; how many lots of two cubes, five

cubes, eight cubes, three cubes, seven cubes. Draw a square; how many sides has this square? how many sides have two squares? Now divide the square into three equal parts, with horizontal lines; divide each equal part into three equal parts, with vertical lines; what is the shape of each one of the parts? how many little squares are there on one side of the large square? how many squares are there on four sides? how many squares are there that do not touch a side of the large square?

"A man had eight geese; one day he sold a quarter of them; how many did he sell? how many had he left? Next day he sold another quarter of his eight geese; how many had he then left? how many had he sold altogether? Two quarters of the geese are the same as what part? What part of his whole flock then had he sold? Take as many pebbles as you have heads, ears, eyes, hands, and noses. There is a story of a giant who had two-mile boots (or boots in which each step he took was two miles long); how many steps must this giant take to go eight miles? How many cakes must I make so that you and John and Anne shall have half a cake each? How many cakes must I make so that six persons shall have half a cake each? that seven persons shall have the same quantity? How many threes are there in nine? fours? fives? etc.

"One loaf weighs half a pound, another weighs a quarter of a pound; how much do both weigh together? Little Philip is eight years old, his sister Mary is five; how much older is he? What is half of two apples? of three apples? four apples? five apples? six apples?

"A pound of new potatoes sometimes costs four cents; how much do two pounds then cost? how much does half a pound cost? how much does a pound and a half cost? Put nine beans in a row; seven beans and how many are nine? five beans and how many are nine?

"A lame horse went along the road two miles an hour; another horse went six miles an hour; how much faster did one go than the other? Draw a picture of a road on the blackboard (a broad line); make a mark across the middle of the line.

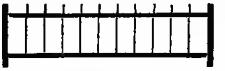
Here is the stable. I start with my horse from the stable in one direction and go five miles; you start and go four miles in the opposite direction; how far should we then be from each other? If I travel six miles an hour and you travel four, in opposite directions, (show on board,) how far would we be apart in half an hour? How far should we be apart if we walked an hour in the same direction?

"How much does a quart of pop-corn cost at three cents a pint? A cat caught a mouse every other day for four days; how many did she catch? If eight eggs cost four cents, how much do two eggs cost? Make nine crosses in one row, nine more in two rows, nine more in three rows; how many threes do you find in nine? How much do I pay for a yard and a half of calico at six cents a yard? Tell me all the sets of two numbers that are equal to nine; to seven; to nine. Tell me what numbers you can separate six into. Tommie Lennen bought four cents' worth of ginger-bread and sold it for one-half more; how much did he sell it for? How many equal numbers are there in nine, two, three, etc.? A boy drove a calf to market at the rate of two miles an hour; how many hours did he spend

in going to market, which was eight miles off? If six be *divided* by two, or divided into twos, how many such parts will be found in it? Two, then, is *contained* in six how many times? For shortness we say, 'two's in six three times;' or, 'two's in six three;' or, 'six divided by two is three.' Twice what is six? Three times what is six? Half six is what? What should I pay the butcher for half a pound of meat, if a pound cost seven cents? I go to market with nine cents in my pocket; I buy two apples at a cent apiece, four eggs at half a cent apiece, and with what is left of my money I buy three peaches; how much do I pay apiece for the peaches?"

Operations with Objects to 10.

"Show me nine marks; add one to them; nine and one are called ten. Show me ten blocks; ten shells; ten shoe-pegs; ten horse-chestnuts; ten pebbles; ten splints; ten boys; ten girls; ten children; ten slates; ten books; ten panes of glass. Make ten triangles; ten squares; ten crosses; ten vertical lines; ten slanting lines; ten horizontal lines. Draw an apple-tree and put ten apples upon it; now put ten birds in the branches. Draw a house with ten windows and half as many chimneys. Draw ten pens, and put ten pigs in each pen. Make a stable with blocks and put ten horses (shells, etc.) in it. Take ten steps; shake hands with ten persons. Draw a fence with ten posts, and ten slats (picket) between each two posts.



"Separate ten cubes into two equal parts; how many are there in each part? How many fingers have you on both hands? on each hand? How many fives of fingers does it take to make ten fingers? Separate ten pebbles into lots of two each; how many such lots can you find? Put out ten shells, — take away three; how many are left? Take away eight; how many are left? I have put out six blocks; give me as many as will make ten blocks."

Give a large number of examples of this kind. First have pupils handle counters, then have them answer by simply seeing.

"Here are how many blocks?" "Ten." "Turn around (teacher takes away four); turn around; how many now? How many did I take away? Shut your eyes (takes away six); open your eyes; how many have I taken away? how many are left? I put down three counters; give me as many as will make ten counters. Shut your eyes; give me ten blocks. Write ten i's; write ten words; ten sentences. Write ten i's in five rows; in two rows. How many weeks are there in two months? How many months are there in six weeks? How many weeks are there in a month and a half? How many weeks are there in three-quarters of a month? How many weeks are there in a month and a quarter? What part of a month is a week? What part of a month is a fortnight? Take ten shells; now find how many twos you can divide them into; how many fives; threes; eights; fours; sixes; etc.

"A woman bought eight cups and saucers; her servant broke two, and her little boy broke three; had she any left? A mother gave eight apples to her eldest daughter, to divide with her two brothers and one sister; how many should each of the four children have?

If only four apples had been given how many should each child have had? how many if two apples had been given? if one had been given? If I were to divide an apple into eight parts, what would each part be called?

"How many pages in a book should you read in a week if you read one in a day? How many if you read half a page a day? Seven pebbles and what are ten? three and what are ten? etc. A farmer had nine acres of wheat, and at harvest-time he reaped three acres a day; how many days did it require to reap all his wheat? How many weeks are there in a month and a week? Take out ten cubes and put them in a row; five cubes and four cubes and what are ten? three cubes and five cubes and what are ten? A wooden pile, or post, was driven three feet into the bottom of a river, two feet more were covered with water, and three feet more were above water; how long was the post from top to bottom? what part of the post was covered with water? If you are five years old now, in how many years will you be eight? Who will be eight in two years? why?

"Put ten counters in a row; put other ten counters in two rows; put other ten counters in three rows; four rows; five rows. Here are four little boxes; now put these ten beans into the boxes in as many ways as you can. Take five splints; if each of these splints were divided into two parts, how many half cubes should you have? A tall man's steps were each a yard long, his little boy's steps were a foot long; how much longer were the father's steps than the son's? If you put two cents into your savings every day, in how many days would you have ten cents in the bank? Supposing that you put in a five cent piece every week, in how many months would you save ten five cent pieces? What must I pay the milkman in a week for a pint of milk every day, except Sunday, when milk is three cents a quart? Tell me all the numbers that are the same as seven. A bureau has five drawers; each drawer has two handles; how many handles are there in the bureau? Find all the sets of two numbers that will make ten. Show the twos of ten; the threes; fours; fives. Find how many ways you can make ten with the blocks. Show me three numbers that are ten; four numbers; five; six; seven; eight. I build here with the blocks five pig-pens; no, you may build them; now how many pigs have I in each pen? None: then I will put three pigs into each of four pens, and one pig into this last pen; how many pigs in all?"

TERM.	ESSENTIALS.	INCIDENTAL AND DIRECTIONS.	SUGGESTIONS AND REFERENCES.
I.	1. **3.**	1 *a*. Ascertain definitely how much of numbers each pupil knows. 1 *b*. **To be known at sight.** 1. The number itself. 2. Every combination of two numbers that makes the number. 3. Every separation of the number into two numbers.	R. Study Grube's method in Indianapolis Manual, Grube's Tables, Grant's Arithmetic for Young Children, Milnes' Primary Arithmetic, Olney's Primary Arithmetic, Franklin Primary Arithmetic, Primary Teacher.
II.	2. **5.**	4. All equal numbers that make the number. 1 *c*. **To be made and seen readily.** 1. All combinations of more than two numbers that equal the number. 2. All separations of the number into more than two numbers. Fractional parts of numbers.	S. 1. **Use objects,** for example, all things and their parts, in the schoolroom, **blocks** and **parts** of **blocks, splints, pebbles, beans, corn, horse chestnuts, money, weights** and **measures,** &c.; **lines** and **forms** on **blackboard, vertical, oblique, horizontal lines, angles, triangles, squares, parallelograms,** &c.; **actions, walking, clapping, stamping,** and other movements of the limbs.
III.	3. **10.**	3. Comparisons of the number with all numbers that have been previously taught. 3 *a*. Pupils to make examples for others to solve,—with objects.	S. 2. The main purpose in teaching number is to train the senses to quick and accurate **seeing, hearing,** and **handling.** Seeing—by presenting objects rapidly and requiring pupils to tell what they see; seeing—hearing—and handling—by the pupils' handling objects under systematic direction. S. 3. **Proceed very slowly.** Much time should be given to those who do not learn easily.

TERM.	ESSENTIALS.	INCIDENTAL AND DIRECTIONS.	SUGGESTIONS AND REFERENCES.
I.	4. **10,** with written signs.	4 *a*. Objects must be used until distinct, easily recalled ideas of numbers are firmly fixed in the mind; that is, until pupils form the habit of instantly recalling mental pictures of things when required to perform examples in concrete numbers. **Until this is done reasoning is impossible.** 4 *b*. Associate the ideas of number already acquired with the figures that represent them.	R. How to Teach, p. 20. S. 3. S. 4. Ideas of number are to be associated with figures in precisely the same way that they have been with their oral signs. See 1 *b*, *c*, D. Prim
II.	5. **15,** oral and written.	1. Let pupils show a number, and the teacher write its name on the blackboard. 2. Let the teacher show a number, and pupils write its name on blackboard or slates. 3. Same with all combinations and separations. 4 *c*. Writing the nine digits and cipher. 1. Teach the writing of one figure at a time. 2. Train pupils to make good figures and arrange them neatly. Teach all the forms of writing numbers, using signs, etc.	S. 5. The writing of figures should be taught in the same way that the forms of letters are. Order, 1, 4, 7, 0, 9, 6, 5, 2, 3, 8. S. 6. **Never allow any careless work to be done.**
III.	6. **20,** oral and written.	6 *a*. Use Grube's table of 10 for reviews. 6 *b*. Roman numerals to XX. 6 *c*. Combinations and separations to be read in the briefest possible way, $3+3$, $2+4$, two 3's, three 2's, $10-4$, to be read as 6 with no more hesitation than in reading the figure 6 itself. 6 *d*. Give many problems in concrete numbers to be solved with objects.	S. 7. Proceed so slowly and thoroughly that pupils will form the **habit** of being accurate in all calculations. **Never allow a mistake to be made if possible.**

TERM.	ESSENTIALS.	INCIDENTAL AND DIRECTIONS.	SUGGESTIONS AND REFERENCES.
I.	TEST RESULTS.	T. R. *a.* Examine thoroughly every step, and step by step all previous work. Build up the weak places. **Do not advance one jot until everything required up to this grade has been done.** T. R. *b.* Use Grube's Table of 20 for reviews.	R. McVicar's Primary Arith. How to Teach, pp. 41, 52, 67, 88. French's Primary Arith. New Primary Object Lessons, Calkins, S. 6. S. 8. The importance of testing results cannot be overestimated.
II.	7. 50.	7 *a.* From 50 to 10,000 the requirements in developing number are,— 1. To know **perfectly** at sight and hearing all multiplications and divisions (without remainders) to 144 (multipliers and divisors not to exceed 12). 2. To read and write perfectly the numbers taught. 3. To calculate **orally** with accuracy and rapidity with numbers that can be easily retained in the mind. 4. To add, subtract, multiply and divide, by writing, all numbers contained in the number to be developed (not using fractions). 5. To apply the above in performing examples in concrete numbers. 7 *b.* Halves, fourths, sixths, eighths, tenths, to be taught with objects only, in the same manner that numbers from 1 to 10 have been taught.	S. 9. Order of development:— 1. Combine, proceeding slowly from the smallest to the largest, three numbers (one place each) as $1+3+2$, $4+3+1$. Then four numbers, five six, seven, &c., using all the forms of writing numbers. 2. Two numbers, one represented by two places and one by one place. First, amounts not to exceed 9; second, amounts to exceed 9, as $15+4$, $21+8$, $19+4$, $25+8$. 3. Two numbers, each represented by two places. First, single amounts not to exceed 9; second, amounts to exceed 9; third, 3, 4, 5, 6 numbers, &c., as $13+15$, $24+33$, $14+18$, $24+16$, $44+66$, $12+13+14$, $24+16+13$, &c. 4. Follow the same plan with numbers of 3 and 4 places. 5. Separate number represented by two places; the subtrahend less than unit figure of minuend; then subtrahend greater than unit figure: $16-4$, $18-5$, $16-7$, $25-6$, &c.

		7 *c.* Use U. S. Money, Liquid, Dry and Time measures in applying numbers. NOTE. Long Division is the easier process, therefore it should be taught first. After pupils become skillful in this, Short Division should *always* be used when the divisor is less than 13. **Great care** should be taken to follow *work done* with the next and **simplest step.** Proceed from beginning to end **without a break.**	6. Separate number of two figures, subtrahend (two places each figure to be less than figures in minuend; then unit figure in subtrahend greater: 25—14, 36—22, 24—15, 20—16, &c. 7. Same with numbers represented by three and four places. 8. Multiplicands from 13 upwards. Products not to exceed the number being developed. First, multipliers less than 10, single products not to exceed 9. Second, single products to exceed 9. Third, multipliers of two places, then three, &c. 9. Dividends, two places, quotients to exceed 12, divisors from 1 to 13. First, without remainders in single divisions. Second, with remainders. Third, dividends, three, four, five places. 10. First,—dividends, two places, divisors from 13 upwards. Second,—dividends of three and four places. This order is suggested simply, trusting that the teacher will discover some better way. S. 10. All difficulties, such as "carrying," "borrowing," dividing by numbers greater than 12, should be **plainly seen** by pupils; then they should be carefully led to discover the best means of overcoming them. S. 11. The fewest words and briefest formulas possible should be used in calculations. Correct and rapid **doing** is the thing required.
III.	8. **100.**	8 *a.* Roman numerals to C. 8 *b.* Thirds, fifths, sevenths, ninths. Change halves to fourths, sixths, eighths; tenths to fifths, &c. 8 *c.* Use inches, feet, yards, rods. Train pupils to estimate distance. Require pupils to use repeatedly meter, liter, and gram.	S. 12. Very little time is needed in teaching Roman numerals. Pupils make them on slate and blackboard. Exercises in recognizing at sight on blackboard; drills in finding chapters.

TERM.	ESSENTIALS.	INCIDENTAL AND DIRECTIONS.	SUGGESTIONS AND REFERENCES.
I.	TEST RESULTS. 9. **200.**	T. R. *a*. See that everything previously taught, is kept fresh by constant use.	R. See T. R. *a*, B Prim. Examine all previous references, and study **the whole course** thoroughly. S. 8.
II.	10. **1000.**	9 *a*. Continue study of fractions with objects. Add and subtract fractions that can be easily changed from one form to another (reduced), with objects, as $\frac{1}{4}+\frac{1}{2}$, $\frac{1}{3}+\frac{1}{6}$, &c.	S. 13. Teach weights and measures with objects. S. 14. Great care must be taken in leading pupils to **understand examples. Train them to form a habit of thoroughly understanding examples before they attempt to perform them.**
III.	11. **10,000.**	9 *b*. Use Avoidupois Weight, Long, Cloth and Surface Measures and Miscellaneous Table, meter, liter, and gram. 11 *a*. Tables of weights and measures used. 11 *b*. **Write and receipt bills of goods. Continue this throughout the course.**	S. 15. A good way to teach tables of weights and measures is, after teaching with objects, to write them upon the blackboard and use them in performing a large number of examples; then erase.

TERM.	ESSENTIALS.	INCIDENTAL AND DIRECTIONS.	SUGGESTIONS AND REFERENCES.
I.	TEST RESULTS. 12. Numeration and Notation.	T. R. *a*. Mental activity, delight in the work, power to comprehend new subjects, and the habit of trying to understand examples, are the best proofs that the previous work has been well done. 12 *a*. Up to this grade the teaching has been purely **elementary.** Pupils have been **seeing** and **doing;** they have acquired clear, **growing** conceptions of numbers and operations with them by constant and systematic exercise. Now they are required to look closer at what they see and do; to describe things and processes (definitions and rules), to discover reasons that make certain processes necessary (principles). This teaching must be **purely objective.**	R. Grant's Arith., Part II.; McVicar's Hand-book of Arith.; White's Manual of Arith.; How to Teach, pp. 67, 88. S. 16. See T. R. *a*, B Prim. If pupils are weak, and full of faults, **begin** the work again, and take time to **repair the poor instruction.** R. See S. 8. S. 17. The main purpose of this teaching is the development of **mental power.** The learning of each problem, definition, rule, or principle, presents an **opportunity** to increase the power of seeing and thinking. The explanation or learning by heart of one of these, **deprives the pupil** of one important means of growth; as if the trainer of gymnasts lifted
II.	13. Addition. 14. Subtraction.	13 *a*. **Use U. S. money, decimals and compound numbers in teaching fundamental rules.** Teach meter, decimeter, dekameter, liter, deciliter, dekaliter, gram, decigram, dekagram. 13 *b*. Addition and subtraction of fractions, using simple examples, with objects (elementary work). 13 *c*. Writing bills, and keeping ordinary simple accounts, measuring wood, carpets, room walls, &c. 13 *d*. Use very few examples exceeding millions.	all the heavy weights **for** (?) his pupils. **Never tell a pupil anything that he can be led to discover for himself.** S. 18. U. S. money, compound numbers, decimals, weights and measures can be made **important aids** if properly used, in teaching and applying the fundamental rules. Simple examples only should be used. S. 19. Have pupils work together at blackboard quite often.
III.	15. Multiplication. 16. Division.	15 *a*. Multiplication of fractions (elementary). 15 *b*. Simple examples in percentage and interest. 15 *c*. Cubic measure, with objects. 16 *a*. Division of fractions (elementary). 16 *b*. Shorten process, by casting equal factors out of dividend and divisor.	S. 20. Walton's Tables are recommended for drills in calculation. R. See 13 *a*, D Gram.; How to Teach, p. 113.

TERM.	ESSENTIALS.	INCIDENTAL AND DIRECTIONS.	SUGGESTIONS AND REFERENCES.
I.	TEST RESULTS. 17. Fractions.	17 *a*. Ideas of fractional numbers. 1. Fractional number less than a unit. 2. Equals or is greater than a unit. 3. A whole and a fractional number. 4. Numeration and notation of fractional numbers. 17 *b*. Reduction. 1. A whole number to a fractional number. Mixed number to a fractional number. 2. Fractional number (?) that equals or exceeds a unit to units. 3. To greater denominators. To less denominators. 4. Fractional numbers of different denominators to the same and lowest denominator.	R. McVicar's Arith.; Felter's Arith.; Milnes' Arith.; Franklin Arith.; Eaton's Arith.; Thomson's Arith.; McVicar's Hand-book; Grant's Arith. Part II.; How to Teach, p. 144; Hagar's Arith.; Olney's Science of Arith.; See T. R. *a*, B Prim., S. 17. S. 21. **Every step objective.** Each step in teaching fractions presents an excellent opportunity to develop mental activity. Use continually **simple,** practical examples. S. 22. After the process of finding the least common denominator has been learned, pupils should be trained to discover it, when possible, without going through a written process.
II.	17. Fractions.	17 *c*. 1. Combinations and separations of fractional numbers. 2. Combination of equal fractional numbers; of equal mixed numbers. 3. Fractional part of a whole number. 4. Fractional part of a fractional number. 5. Shorten process (4) by rejecting equal factors in numerators and denominators. 6. Separation of a whole number into fractional numbers. 7. Separation of a fractional number into equal parts.	S. 23. The key to understanding how to combine and separate fractional numbers is found by changing them to common denominators, **and proceeding precisely as in whole numbers.** How many thirds in $\frac{1}{2}$? $\frac{1}{2} \div \frac{1}{3}$. $\frac{3}{6} \div \frac{2}{6}$. $\frac{3}{2} = 1\frac{1}{2}$.

		8. Separation of a fractional number into equal fractional numbers; measuring one fractional number by another. 17 *d*. Use U. S. money, weights and measures in teaching fractional numbers. See 13 *a*, I., D Gram.	NOTE. Write the problem upon the blackboard; question pupils until they all understand it; then call upon one of the class to take the first step — perform it — call upon another for the second step, etc. Be sure that the dullest have the best opportunities to try.
III.	18. Decimal Fractions.	18 *a*. Order of steps: — 1. Numeration and notation. 2. Reduce common fractions to decimal fractions; decimal to common fractions. 3. Addition and Subtraction. 4. Multiplication. 5. Division. 18 *b*. Simple examples in percentage and interest. S. 22. 18 *c*. Use few decimals that are less than ten-thousandths and none that are less than millionths.	R. How to Teach, p. 157. S. 24. Pupils who have learned common fractions have nothing new to learn in decimals except forms of writing. "Pointing off" can be learned in the same way that punctuation is learned, — by correct practice.

TERM.	ESSENTIALS.	INCIDENTAL AND DIRECTIONS.	SUGGESTIONS AND REFERENCES.
I.	TEST RESULTS. 19. Compound Numbers.	19 *a*. All the work in compound numbers is limited to that which is practiced in common life. Weights and measures not commonly used are not to be learned. Very little of addition, subtraction, multiplication and division is to be taught. 19 *b*. 1. Reduction Descending. 2. Reduction Ascending. 3. Weights. 4. Linear measure. 5. Surface measure.	R. Felter's Arith.; White's Arith.; How to Teach, p. 180. Understand thoroughly the whole of this course of study before attempting to test results. S. 7. S. 25. All commonly used tables should be thoroughly known. Not used, dram (av. weight), roods; barrel, hogshead (not often). S. 15.
II.	19. Compound Numbers.	6. Solid measure. 7. Dry and liquid measure. (Omit beer measure.) 8. Circular measure. Questions in the difference of latitude and longitude. 9. Time. Preparation for the study of interest. 10. Miscellaneous table. 11. Money. Few examples, showing difference between U. S. money and English, French, and German moneys. 12. Fractional compound numbers, limited to — a. Common fractions to integers of lower denominations. b. Decimals to integers of lower denominations. c. Compound numbers to fractional numbers of a higher denomination. d. Compound numbers to decimals of a higher denomination. e. Metric system. Teach with objects.	S. 26. A large number of examples in measurements of surfaces and solids, as floors, carpets, ceilings, wood, stone, timber, &c. Second and third powers of numbers. Roots that are easily found. S. 27. The metric system should be compared with the cumbrous system, or lack of system, now in use. The miter, liter and gram should be made familiar objects to pupils. The simple principles should be learned.

III.	20. Percentage.	20 *a*. 1. **Base** and **rate** being given, to find percentage. 2. **Percentage** and **base** being given, to find rate. 3. **Percentage** and **rate** being given, to find base. 4. Amount and rate being given, to find base. 5. Apply the above cases in Brokerage, Commission, Profit and Loss. 20 *b*. Interest (percentage involving time). 1. Base, rate and time being given, to find the interest or amount. Apply in Bank and Commercial Discount. 2. The interest, or amount, and the rate and time being given, to find the base. Apply in Present Worth, True Discount.	R. How to Teach, p. 206; Model Arith. S. 28. **The work in Percentage is limited strictly to common business affairs.** Those methods are to be used which are the most generally applicable. Nothing is to be done with business methods and forms that are obsolete. S. 29. **To be known thoroughly.** All commonly used terms and written forms, such as notes, receipts, &c. S. 30. **Only one method is to be taught in casting interest.** That should be the simplest. S. 28.

EIGHTH YEAR. A GRAMMAR.

TERM.	ESSENTIALS.	INCIDENTAL AND DIRECTIONS.	SUGGESTIONS AND REFERENCES.
I.	TEST RESULTS. 20. Percentage.	3. The base, interest and time being given, to find the rate. 4. The base, rate and interest being given, to find the time. 20 *c*. Compound Interest. Limited to a few examples.	R. S 16, 17.
II.	20. Percentage.	20 *d*. Partial Payments. 20 *e*. Equation of Payments.	
III.	21. Evolution.	21 *a*. Square Root. 21 *b*. Cube Root.	S. 31. To be taught with objects. Give simple examples.

A COURSE OF ELEMENTARY LESSONS ON THE FORM OF BODIES.

By Albert G. Boyden, Principal of the State Normal School, Bridgewater, Mass.

I. The **object of these lessons** is threefold: 1. To train the pupil **to observe the form** of bodies; 2. To **acquire definite ideas of the form** of bodies; 3. To make the **correct expression of these ideas.**

II. The **principles to guide the teaching** are,—1. Present the body **first as a whole,** then proceed to its parts; 2. **Each pupil must observe** the body whose form is to be learned; 3. Ideas of form are **originally acquired only from the body** through the senses of sight and touch; 4. These elementary ideas should be taught so as to prepare the mind for the scientific study of form.

III. The **method of the teaching** is,—1. Present the body to **each pupil; 2. Lead the pupil by definite questions** to **get the idea from the body; 3. Lead the pupil** to the **correct** expression of the idea both in speech and writing.

Note. **Form** is one of the **qualities** of every material object. In these lessons the pupil is to study the **form of the body,** not **form** apart from any body, as is done in Geometry. Hence a **spherical body,** which is a **portion of matter** perfectly round, is presented to the child instead of a **sphere,** which is a **portion of space** perfectly round.

WHAT IS TO BE TAUGHT.	DIRECTIONS.	SUGGESTIONS.
1. To distinguish and name the form of the body as a whole. **Spherical** body. **Cylindrical** body. **Cubical** body.	Take the bodies in the order indicated in the topics, because this is proceeding from the simplest form to the more complex forms.	Use wooden blocks with a smooth surface and well-defined edges for illustration. Teacher should have three or four of each kind, large enough for every pupil in class to observe. Let pupil select each kind from the miscellaneous group after getting the idea from the single object.
2. **Qualities** of these bodies. Spherical body — round, perfectly round. Cylindrical body — round one way, ends plane, called bases. Cubical body — faces all plane.	Observe the spherical body alone; first that it is round, then that it is perfectly round. **Compare** cylindrical body with the spherical body and note the difference. Teach that the face of a body is **the part of the outside** turned **toward the eye.** Then that the faces are all flat or plane.	Have the pupils find other bodies having these forms,—as marbles, oranges, grapes, shot; lead pencils, stove-pipe; pieces of cake, pieces of soap, some boxes.

3. **Surface.** — Plane surface; curved surface. Spherical body — has a curved surface. Cylindrical body — has curved and plane surface. Cubical body — has all the surface plane.	Teach that the **surface is the outside of the body.** Teach the kinds of surface, plane and curved, with the bodies. Then apply these ideas to these bodies.	Have the pupils tell what kind of surface other bodies have, bodies in the room, and bodies not in sight.
4. The **parts** of these bodies. Spherical body, — surface one, no parts. Cylindrical body — **curved surface,** one. **faces,** plane, two. **edges,** curved, two. Cubical body — **faces,** plane. **edges,** straight. **corners or angles.** **number** and **relative position** of parts.	Teach the part, — its distinguishing quality, the number of parts. In the cubical body teach the relative position of parts with the number, as one face above, one below, and four **lateral** faces. Notice edges and corners in same way.	Have the pupils tell the number and relative position of the faces or walls, edges, and corners of the rooms, of the school-house; and other buildings in sight.
5. **Line. Straight** line. **Curved** line. Spherical body — has no lines. Cylindrical body — each face has a **curved** side. Cubical body — each face has **straight** sides.	Teach that **a line is the limit or boundary of** a face or a surface. Teach the kinds of lines, straight and curved, from the faces of the cubical body and the cylindrical body. Then apply these ideas to the bodies.	Confine the use of the word "side" in these lessons to a line. Do not call the face of a body a side. Each term should have only one meaning in this study. Have pupils find straight and curved lines on other bodies.
6. **Positions** of one line. Vertical position. Horizontal position. Inclined position.	Teach that the vertical position of a line is the position of a plumb line, using a plumb line. Teach that the horizontal position of a line is the position of a line on the surface of a level floor, or on the surface of a level table, or on the surface of water at rest in a pan. Teach that the inclined position of a line is any position between a vertical and a horizontal position, or it is the position in which a line leans toward a horizontal position.	Use the sides of the lateral face of the cubical body in teaching these positions, for the vertical and horizontal positions let the face of the cubical body rest on the table and note position of the sides of the lateral face. Then let the edge of the body rest on the table, and note the positions of the sides of the lateral face, with reference to the table surface. Let pupils make lines on slate in these positions.

7. **Relative position** of two lines. Two lines **parallel** to each other. Two lines **inclined** to each other. Two lines **perpendicular** to each other.	Teach that two lines are parallel to each other when they have the same direction. Teach that two lines are inclined to each other when one leans either toward or from the other. Teach that two lines are perpendicular to each other when they differ in direction and one neither leans toward nor from the other.	Use the opposite sides of the face of the cubical body to teach that two lines are parallel. Use the two adjacent sides of the face to teach that two lines are perpendicular to each other. Use the adjacent sides of a face which is an oblique parallelogram to teach that two lines are inclined. Speak of two lines as being parallel, not of two parallel lines.
8. **Angle. Right** angle. **Oblique** angle — { **acute** angles. / obtuse angles.	Teach that the two sides of the face of the cubical body extending from the same point in different directions form an angle. Teach that when the sides of the angle are perpendicular to each other the angle is a right angle, when they are inclined to each other the angle is oblique. Teach the varieties of oblique angles.	Use the two sides of a face which is an oblique angled parallelogram to teach the oblique angle and its varieties. The angle comes as one part of the face of the cubical body.
9. The **Faces** of these bodies. Their **number** and the **qualities** of their **parts.** Spherical body — surface one, no faces. Cylindrical body — two equal plane faces, perfectly round or circular, one curved face. Cubical body — faces have four straight, equal sides, four right angles; faces are square.	Measure the faces of cylindrical body by a piece of card of size of one face to show that they are equal. Measure the sides and angles of the faces of the cubical body to show that they are equal.	Use a right angle cut from a piece of card in measuring the angles. Let the pupils make the faces of the cylindri- body and the cubical body on the slate.
10. **Rectangular** bodies. All plane faces as **wholes.** **Parts** — faces, edges, corners. **Faces** — have four straight sides, four right-angles, opposite sides parallel; all the faces are rectangular.	The form of the body is determined by the faces which bound or limit it. Hence in teaching the form of a new class of bodies observe the body first in its faces as a whole, then its parts, then the number and qualities of the faces.	Use a number of rectangular blocks of different sizes. Have pupils find other rectangular bodies and note their parts. Explain the word "rectangular" as applied to the face, — as a face having all its angles **right** angles.

11. Triangular bodies. **As wholes,** all plane faces. **Parts,** faces, edges, corners. **Faces,** — the two bases have three sides, three angles; the bases are triangular. The lateral faces are rectangular.	Observe the body as a whole to teach the kind of faces which bound it. Teach the number and relative position of the parts. Teach the qualities of the parts.	Use a number of triangular blocks of different sizes. Have pupils find other triangular bodies and notice their parts. Explain the word "triangular" as applied to the face as meaning that the face has three angles.
12. Polyedral bodies. **As wholes,** all plane faces. **Parts,** faces, edges, corners; number and relative position of parts. **Faces,** — bases have more than four sides and angles; bases are called **polygons.** The lateral faces are rectangular.	Teach the polyedral body whose bases have three sides. Those which have six sides to the base. Teach in the order stated in 11.	Have a number of blocks of each kind of different sizes. Let the pupils make the faces of the bodies on the slate. Explain the word "polyedral" as meaning a face having **more than four sides.** Explain the word "polygon" as meaning a face having **more than four angles.**
13. Pyramidal bodies. **As wholes,** all plane faces. **Parts,** faces, edges, corners. **Faces,** — base may have any number of straight sides; base is a **rectilinear figure.** Lateral faces are triangular.	Teach pyramidal body whose base has three sides; then one whose base has four sides; then one whose base has five sides.	Use a number of pyramidal blocks of different sizes. If you have not blocks make bodies of cardboard. Make pyramidal bodies of cardboard if you have not blocks enough of the kind, or make them from turnips or potatoes. Explain the words "rectilinear figure" as meaning a face having straight sides.
14. Conical bodies. As wholes — surface covered and plane. **Parts,** curved surface, circular base, edge, vertex.	Compare conical body with pyramidal body and notice the difference in surface and parts.	Use a number of conical blocks, or other conical bodies of different sizes.

15. Spheroidal bodies. As wholes — not perfectly round, but somewhat like a spherical body. **A Prolate** spheroidal body. As a whole — elongated. **An Oblate** spheroidal body. As a whole — flattened. **Hemispherical** bodies. Half a spherical body. Parts, circular base, edges, spherical face. **Conoidal** bodies. As a whole — like a conical body, but tapering in a **curved line** instead of a **straight line,** as the conical body tapers. **Parts,** circular base, edge, curved face. **Egg-shaped** bodies. As a whole — like an egg.	Teach the spheroidal and hemispherical bodies by comparison with the spherical body. Teach the conoidal body by comparison of it with a conical body. Teach the egg-shaped body by comparison with an egg. Teach the prolate and oblate spheroidal bodies in such a way as to show the meaning of the words "prolate" and "oblate."	Use some spheroidal bodies which are nearly spherical and others more prolate or oblate. Explain the terms "spheroidal" "hemispherical," and "conoidal." Have pupils find other spheroidal bodies, as a watermelon, a peach, a lemon, an onion, a white turnip. Have pupils find other conoidal bodies, as a hay-stack, an acorn, a pine-apple, some berries. Use several bodies of each kind of different sizes.
16. The form of the **different plane faces** of bodies. **(1) Circles.** As a **whole** — any face perfectly round. Parts. — Circumference — the line which bounds the circles.	Teach the circle by using the bases of the cylinder, the semi-circle by using the base of half a cylinder, and the quadrant by using the base of quarter of a cylinder. Teach the other parts by making the point and lines on the base of the cylinder.	Have the pupils find other bodies which have faces that are circles or parts of circles. Have the pupils draw circles and parts of circles on slate and paper; and cut out the circles drawn on paper. The **circular** face came in connection with the cylindrical body; now the circle and its parts is to be taught so that it may be used whenever needed.

Centre — the point equally distant from every part of the circumference. Radius — the straight line from the centre to the circumference. Diameter — the straight line through the centre having each end in the circumference. Semi-circle — half a circle. Has one straight side, one curved side. Quadrant — quarter of a circle. Has two straight sides and one curved side. Crescent — a part of a circle which has two curved sides.	Teach the crescent by comparison with the semi-circle. It differs from the semi-circle by having a curved side instead of the straight side.	The different plane faces of bodies should be taught in such a way as to show two things, namely, the **qualities of the face,** and the **number of bodies** which have such faces; that is the **idea** of the class and the number of objects in the class. Hence many objects of each kind are necessary to make these two ideas plain.
(2) Triangles. As a whole — any face which has three straight sides and three angles. Parts — sides and angles. An **Equilateral** triangle — a triangle which has its sides equal. An **Isosceles** triangle — a triangle which has only two sides equal. A **Scalene** triangle — a triangle which has no two sides equal. An **Equiangular** triangle — a triangle which has all its angles equal. A **Right-angled** triangle — a triangle which has one right angle. An **Oblique-angled** triangle — a triangle which has all its angles oblique.	Teach the triangle by using the triangular faces of triangular bodies, such as will show the different variety of triangles in their faces. Teach in such a way as to show the meaning of the name applied to the different varieties of triangles.	The kindergarten blocks furnish good illustrations. Let the pupil draw all the varieties of triangles on their slates and on the board, and then have them tell what they are and why they name them as they do. The gonigraph is very useful in illustrating the varieties of form of these plane faces. Triangular bodies cut from thick pasteboard make good illustrations. Do not call these bodies triangles. Only the face is a triangle.

(3) Quadrilaterals. As a whole — any face which has four straight sides and four angles. Parts — sides and angles. A **Square** — a quadrilateral which has four right angles and four equal sides. A **Rectangle** — a quadrilateral which has four right angles, two long sides and two short sides. A **Rhomb** — a quadrilateral which has four oblique angles and four equal sides. A **Rhomboid** — a quadrilateral which has four oblique angles, two long sides, and two short sides. A **Parallelogram** — a quadrilateral which has its opposite sides parallel. A **Trapezoid** — a quadrilateral which has only two sides parallel. A **Trapezium** — a quadrilateral which has no two sides parallel.	Teach the quadrilaterals by using such bodies as will show the different varieties of quadrilaterals in their faces. The faces of cubical and rectangular bodies will illustrate the square and rectangle. Teach rhomb and rhomboid by using bodies which have bases of this form. Teach parallelogram by using all the bodies in which the opposite sides of the faces are parallel. Teach trapezoid and trapezium by using bodies which have bases of this form.	Let the pupils draw and name as directed above. Use the gonigraph. Have many examples of each kind so as to give the pupil an idea of the **extent** of the class, as well as of the **form.**
(4) Polygons. As a whole — any face which has more than four straight sides and more than four angles. Parts — sides and angles. A **Pentagon** — a polygon which has five sides and five angles. A **Hexagon** — a polygon which has six sides and six angles. An **Octagon** — a polygon which has eight sides and eight angles.	Teach the different polygons by using the faces of the polyedral bodies which show them. Teach in such a way as to explain the meaning of the name applied to each form.	Let the pupils draw and name the polygons. Use the gonigraph. Other polygons may be taught if needed. Polyedral bodies cut from thick pasteboard make good illustrations.